Stat 610 Homework 8

Thursday, November 16, 11:59pm

Assignment

1. The beta distribution Beta(α, β) is a continuous distribution that takes values in $[0, 1]$. The probability density function for Beta(α, β) is

$$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{\Gamma(\alpha)\Gamma(\beta)/\Gamma(\alpha+\beta)}.$$

   Implement an accept-reject algorithm where the proposal distribution is uniform on $[0, 1]$ and the target distribution is (a) Beta$(2, 2)$ or (b) Beta$(10, 10)$.

   In your version of the accept-reject algorithm, include code that allows you to monitor how many times you had to propose a value before one was accepted.

   For each type of beta distribution, plot a histogram of the accepted values you obtained from the algorithm and report the average number of proposals per accepted sample. Write a small description of what accounts for the difference in the number of accepted proposals betwen (a) and (b).

   Some notes that might be helpful:

   - Notice that when $\alpha = \beta$, the pdf is symmetric.
   - If they are both also greater than 1, the pdf takes its maximum value at $1/2$.
   - The Γ function is a generalization of the factorial function, and is available in R as `gamma`.
   - You can use the `hist` function to plot a histogram.

2. (a) Make a Monte Carlo estimate of the integral $\int_0^1 \cos(\pi x/2)\, dx$, noticing that the integral should be equivalent to $E[\cos(\pi U/2)]$, where U is a random variable that is uniform on the interval $[0, 1]$.

   (b) Make an importance sampling estimate of the integral in (a) using the importance distribution Beta$(1, 1.5)$. (This corresponds to the importance distribution $h(x) = \Gamma(2.5)(1 - x)^(.5)/\Gamma(1.5)$, which you can also get in R as `dbeta(x, 1, 1.5)`. You can sample from this beta distribution using `rbeta(n, 1, 1.5)`.)

   (c) Estimate the variances of each estimate and compare. What accounts for the difference?

Submission parameters

Submit two files:

– A pdf writeup containing your plots and answers to the questions.
– A file containing the code you used.